ESCUELA TÉCNICA SUPERIOR DE INGENIERÍA

DMA-GITI-101, DMA-GITT-101 Algebra and Geometry

SEMESTER: Annual (Fall and Spring)

CREDITS: 9 ECTS (3 hrs. per week)

LANGUAGE: Spanish

DEGREES: **GITI,** GITT

Course overview

This course is an introduction to Linear Algebra and Differential Geometry. It focuses on providing the basic tools from the linear algebra and differential geometry to solve practical problems, and in showing some of their applications in the engineering field. All the contents will be analyzed including several examples taken from the real life or other sciences like physics, economics, etc. The theoretical classes complement each other with practical sessions in the laboratory where the problems are solved using the computer.

Prerequisites

Basic knowledge of Algebra, Geometry and Real Analysis is required for attending this course.

Course contents

Theory:

Part I

1. Matrices. Elementary matrix operations. Rank of a matrix. Inverse matrix. Elementary square matrices.
2. Introduction to Vector Spaces. Vector subspaces. Linear independence and dependence. Basis and dimension. Coordinates and change of basis.
3. Determinants and Linear Systems. Properties of determinants. Gauss elimination. Numerical methods: Triangular factorization and iterative methods.
4. Introduction to Linear Transformations. Kernel and range. The associated matrix to a linear transformation. Composition of linear transformations.
5. Eigenvalues and Eigenvectors. The Jordan Canonical Form. Characteristic Polynomial. Invariant subspaces. Diagonalization. Cayley-Hamilton theorem. Approximation of eigenvalues and eigenvectors.
6. Inner Product Spaces. Inner products and norms. Orthogonal projections. Gram-Schmidt process. Least squares. Diagonalization of symmetric matrices.

Part II

7. Affine Spaces. Definition and properties. Affine subspaces.
8. Orthogonal transformations and Affine transformations. Introduction and properties. Movements on affine spaces.
9. Introduction to curves. Length of an arc of curve. The moving frame. The Frenet formulas. The Helix. Evolutes and involutes. Envelope of a family of curves in the plane.
10. Introduction to surfaces. The tangent plane. Revolution surfaces. Ruled surfaces. Curves on a surface.

Laboratory:

There will be seven 1-hour sessions during the course, between the third and the last lecture week.

P1. Introduction to *Maxima/Matlab* software. Matrices and determinants.
P2. Problem resolution in vector spaces.
P3. Numerical methods for solving linear system: Jacobi, Gauss-Seidel and Relaxation methods.
P4. Modelling problems with linear transformations
P5. Numerical methods for computing eigenvalues and eigenvectors. The Power and Inverse Power methods.
P6. Euclidean vector spaces.
P7. Orthogonal and affine transformations. Geometric problems in A2 and A3.

Textbooks

- De la Villa, A. Problemas de Álgebra con esquemas teóricos. Ed. CLAGSA. Madrid 2010.
- López de la Rica, A. y De la Villa, A. Geometría Diferencial. Ed. CLAGSA. Madrid 1997.

Grading

The overall grade by term is obtained as follows:

- Final term exam 50%.
- Mid-term exam (1.5-hour long) 25%.
- Two additional short term exams 15%.
- Lab term exam 10%.
- The final second term exam will only cover the contents taught in the second term, if the first term overall grade is at least 4 over 10. In other case, it will cover all the contents of the course.

The following conditions must be accomplished to pass the course:

- If the first term overall grade was at least 4, then the second term overall grade must be at least 4 over 10 and the average of both overall grades (first and second terms) must be at least 5 over 10.
- If the first term overall grade was less than 4, then the second term overall grade must be at least 5 over 10.