DMA-SAP-437 NUMERICAL ANALYSIS

SEMESTER: Spring

CREDITS: 6 ECTS (4 hrs. per week)

LANGUAGE: English

DEGREES: SAPIENS program

Course overview

This course covers an introduction to programming and problem-solving with *Matlab*, and an introduction to the most extended numerical techniques in the engineering field to solve a great variety of technical problems. Many phenomena arising in engineering, physical and biological sciences can be described using mathematical models which frequently cannot be solved explicitly. A common approach is using a numerical method to find an approximate solution.

The main targets of this course are the following. On the one hand, to provide the basic programming skills with *Matlab*. On the other hand, to provide a description about of the most important numerical methods to approximate solutions of linear systems, nonlinear equations and nonlinear systems of equations, to optimize and interpolate functions, to approximate integrals and solutions of initial value problems and boundary value problems for ODEs and PDEs. In each numerical method the algorithm, accuracy, conditioning, stability and rate of convergence will be analyzed and the implementation with the software *Matlab* will be carried out.

Prerequisites

Basic knowledge of calculus and linear algebra

Course contents and methodology

Methodology

Lecture, solving calculation problems during exercises and lab sessions.

Contents

0. INTRODUCTION TO PROGRAMMING AND PROBLEM SOLVING WITH *MATLAB.* The *MATLAB* environment. Data structures. Input/Output. Logical flow control statements (decision and looping). User-defined functions.

1. INTRODUCTION TO SCIENTIFIC COMPUTING. Errors, conditioning, sensitivity, accuracy and stability. Floating point.

2. SYSTEMS OF LINEAR EQUATIONS. Matrix norms, condition number and conditioning. Back-substitution and forward-substitution. Gaussian elimination. LU factorization. Reduced LU factorization. The Sherman-Morrison formula.

3. LINEAR LEAST SQUARES. Concept, properties and geometrical interpretation. Orthogonal projectors. Sensitivity and conditioning of Least Squares. Least squares by transformation. Orthogonal matrices. QR factorizations. Singular Value Decomposition of a matrix.

4. EIGENVALUE PROBLEMS. Eigenvalue and eigenvector concepts. Multiplicity of an eigenvalue and diagonalizable matrices. Sensitivity. The Schur factorization. The Iteration Power. Convergence of Power Iteration. Rayleigh Quotient Iteration. Multiple eigenvalues and orthogonal iterations. Krylov space methods.

5. NONLINEAR EQUATIONS. Conditioning of root finding. Iterative procedures and convergence rates. The Bisection method. The Fixed Point Iteration. Newton's method. Secant method. Methods for solving systems of nonlinear equations.

6. OPTIMIZATION. Objective function, minimizers and maximizers. Optimality conditions. Conditioning. Unconstrained optimization in one dimension: Golden section search and Newton's method. Unconstrained optimization for several variables: Steepest Descent and Newton's method in n-dimension. Nonlinear Least Squares. The Gauss-Newton method. Constrained optimization and Lagrange Multipliers. Inequality-Constrained Optimization and Karush-Kuhn-Tucker conditions.

7. INTERPOLATION. Statement of the problem. Modes and nodes. Lagrange interpolation. Newton interpolation. Orthogonal polynomials. Chebyshev interpolation. Piecewise linear interpolation and piecewise cubic interpolation (Splines)

8. NUMERICAL INTEGRATION AND DIFFERENTIATION. Statement of the problem and conditioning. Quadrature methods: Newton-Cotes quadrature and Clenshaw-Curtis quadrature. Accuracy and stability. Composite Quadrature. Gaussian quadrature. Numerical differentiation. Finite differences. Richardson extrapolation.

9. INITIAL VALUE PROBLEMS FOR ODEs. Problem statement and basic definitions about ODEs. Existence, uniqueness, conditioning and stability. Explicit and implicit methods. Euler's method. Local and global error. Predictor-Corrector methods. The Runge-Kutta methods. Single-step and multi-stage methods. Multi-step Adams methods.

10. BOUNDARY VALUE PROBLEMS FOR ODEs. Basic definitions about BVP. Existence, uniqueness and conditioning. Shooting method. Finite Difference method. Collocation method. Galerkin-Finite Element method.

11. PARTIAL DIFFERENTIAL EQUATIONS AND SPARSE LINEAR ALGEBRA. Sparse linear systems. Stationary iterative methods. The Conjugate Gradient method. Introduction to PDEs. Hyperbolic, parabolic and elliptic PDEs. Basic ideas and examples.

12. AN INTRODUCTION TO THE FOURIER TRANSFORM AND FAST FOURIER TRANSFORM

About the required texts:

- Lesson 0 follows the scheme of [1].
- Lessons 1-12 follow the chapter scheme of [2] in name and order.

Textbooks

- [1] *MATLAB*: *A Practical Introduction to Programming and Problem-Solving.* Attaway, Stormy. (for Lesson 0)

- [2] *Scientific Computing. An Introductory Survey.* Second Edition, 2002. Michael T. Heath. McGraw Hill. (for Lessons 1-12)

Grading

The assessment of this subject will involve theoretical and practical questions. The overall grade will be obtained as follows:

- One mid-term exam about lesson 0 (10%)
- Three mid-term exams about lessons 1-12 (10%, 20% and 30%, respectively).
- Homework and final term lab group assignment with the software *Matlab* (30%)

The students whose grades are less than 5 or those who want to improve their previous grades will do a final exam the last day of the course. Their final grade will be the maximum between the grade obtained in this final exam and the result of computing 70% of the final exam and 30% of the homework.

The students who fail the course will have the chance to do an extraordinary exam. The grade obtained in this exam will be their definitive grade.

The exams are all closed notebook and closed textbook. The course will not be graded on a curve, i.e., there is no bound on the numbers A's, B's, C's, etc.